Introduction to Management Science

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# Problem Introduction:

Pak Arab Refinery Limited (PARCO) is a fully integrated energy company and is one of the largest companies in Pakistan's corporate sector. It has three major services to offer – refining, transportation and marketing. In this report, we have focused on the transportation service. The goal is to minimize cost of transportation while meeting all the requirements at the various sites that PARCO supplies fuel to. PARCO transports fuel from Sources (PPL Hala, JJVL Jamshoro, OGDCL Bobi and Kunnar) to these destinations:

1. Karachi
2. Lahore
3. Faisalabad
4. Rawalpindi
5. Peshawar
6. Multan
7. Hyderabad
8. Islamabad
9. Sargodha
10. Sheikhupura
11. Rahim Yar Khan
12. Jhang
13. Dera Ghazi Khan
14. Gujrat
15. Sahiwal

In this problem, we find an optimal route via which the dispatched fuel must be transported to the final centers, so that the costs incurred are minimum.

# Mathematical Formulation

The objective function is to minimize the total distance to minimize the costs associated. Hence, the objective function is:

* MIN: ­ 748x12 + 889x23 + 357x24 + 0.7x25 + 252x26 + 412x27 + 695x28 + 5.5x39 + 1253.5x310 + 1156.1x311 + 1437.7x312 + 1595.6x313 + 903.8x314 + 171.8x315 + 1456.2x316 + 1220x317 + 1244.1x318 + 658.7x319 + 1091x320 + 848x321 + 1354.2x322 + 1074.4x323 + 512.5x49 + 792.6x410 + 695.3x411 + 973x412 + 885x413 + 425x414 + 360.4x415 + 991.5x416 + 759.2x417 + 783.2x418 + 204.1x419 + 572.6x420 + 339x421 + 893.3x422 + 613.6x423 + 893.8x59 + 394.6x510 + 297.3x511 + 575x512 + 732.9x513 + 58.9x514 + 741.7x515 + 593.5x516 + 361.2x517 + 385.2x518 + 259.3x519 + 206.5x520 + 49.4x521 + 495.4x522 + 238.3x523 + 1075x69 + 170x610 + 101x611 + 400x612 + 588x613 + 181x614 + 923x615 + 414x616 + 185x617 + 199x618 + 428x619 + 117x620 + 276x621 + 309x622 + 5x623 + 1245x79 + 65.7x710 + 139.3x711 + 311.7x712 + 471.7x713 + 338.5x714 + 1090x715 + 328.4x716 + 141.5x717 + 8.4x718 + 605.9x719 + 205.4x720 + 431.3x721 + 130.7x722 + 2016.1x723 + 1580.8x89 + 512.6x810 + 457.9x811 + 184.6x812 + 19.9x813 + 676x814 + 1441x815 + 180x816 + 378x817 + 463x818 + 946x819 + 525x820 + 523x821 + 352x822 + 555x823;

*Where XIJ denotes the amount of fuel, in metric tons, supplied from node I to node J*

## Constraints:

* -x12 >= -394;
* x12 - x23 - x24 - x25 - x26 - x27 -x28 >= 5;
* x23 >= 64.833;
* x24 >= 64.833;
* x25 >= 64.833;
* x26 >= 64.833;
* x27 >= 64.833;
* x28 >= 64.833;
* x23 - x39 - x310 - x311 - x312 - x313 - x314 - x315 - x316 - x317 - x318 - x319 - x320 - x321 - x322 - x323 >=0;
* x24 - x49 - x410 - x411 - x412 - x413 - x414 - x415 - x416 - x417 - x418 - x419 - x420 - x421 - x422 - x423 >=0;
* x25 - x59 - x510 - x511 - x512 - x513 - x514 - x515 - x516 - x517 - x518 - x519 - x520 - x521 - x522 - x523 >=0;
* x26 - x69 - x610 - x611 - x612 - x613 - x614 - x615 - x616 - x617 - x618 - x619 - x620 - x621 - x622 - x623 >=0;
* x27 - x79 - x710 - x711 - x712 - x713 - x714 - x715 - x716 - x717 - x718 - x719 - x720 - x721 - x722 - x723 >=0;
* x28 - x89 - x810 - x811 - x812 - x813 - x814 - x815 - x816 - x817 - x818 - x819 - x820 - x821 - x822 - x823 >=0;
* x39 + x49 +x59 + x69 +x79 + x89 >= 154.887;
* x310 + x410 +x510 + x610 +x710 + x810 >= 91.101;
* x311 + x411 +x511 + x611 +x711 + x811 >= 26.24;
* x312 + x412 +x512 + x612 +x712 + x812 >= 17.18;
* x313 + x413 +x513 + x613 +x713 + x813 >= 16.1;
* x314 + x414 +x514 + x614 +x714 + x814 >=15.326;
* x315 + x415 +x515 + x615 +x715 + x815 >= 14.2;
* x316 + x416 +x516 + x616 +x716 + x816 >= 8.268;
* x317 + x417 +x517 + x617 +x717 + x817 >= 5.403;
* x318 + x418 +x518 + x618 +x718 + x818 >= 3.874;
* x319 + x419 +x519 + x619 +x719 + x819 >= 3.442;
* x320 + x420 +x520 + x620 +x720 + x820 >= 3.391;
* x321 + x421 +x521 + x621 +x721 + x821 >= 3.267;
* x322 + x422 +x522 + x622 +x722 + x822 >= 3.198;
* x323 + x423 +x523 + x623 +x723 + x823 >= 3.19;

# The Model

Our model is a transshipment problem with a total of 23 nodes:

Total supply nodes = 1

Total intermediate nodes = 7

Total demand nodes= 15

Nodes represented by,

Supply nodes: 1

Intermediate nodes: 2, 3 ,4, 5, 6, 7, 8

Demand nodes: 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23

We gathered data from PARCO website, there were a few details that were missing. Those were acquired through a contact at PARCO. Google Maps was used to find the distances between supply nodes and intermediate nodes and then intermediate nodes and demand nodes.

Decision variables are the quantity of fuel being transported for every possible route.

Constraints are the supply, demand and storage capacity of each node

Supply Constraint:

* -x12 >= -394;

Storage / Intermediate Constraints:

* x12 - x23 - x24 - x25 - x26 - x27 -x28 >= 5;
* x23 >= 64.833;
* x24 >= 64.833;
* x25 >= 64.833;
* x26 >= 64.833;
* x27 >= 64.833;
* x28 >= 64.833;
* x23 - x39 - x310 - x311 - x312 - x313 - x314 - x315 - x316 - x317 - x318 - x319 - x320 - x321 - x322 - x323 >=0;
* x24 - x49 - x410 - x411 - x412 - x413 - x414 - x415 - x416 - x417 - x418 - x419 - x420 - x421 - x422 - x423 >=0;
* x25 - x59 - x510 - x511 - x512 - x513 - x514 - x515 - x516 - x517 - x518 - x519 - x520 - x521 - x522 - x523 >=0;
* x26 - x69 - x610 - x611 - x612 - x613 - x614 - x615 - x616 - x617 - x618 - x619 - x620 - x621 - x622 - x623 >=0;
* x27 - x79 - x710 - x711 - x712 - x713 - x714 - x715 - x716 - x717 - x718 - x719 - x720 - x721 - x722 - x723 >=0;
* x28 - x89 - x810 - x811 - x812 - x813 - x814 - x815 - x816 - x817 - x818 - x819 - x820 - x821 - x822 - x823 >=0;

Demand Constraints:

* x39 + x49 +x59 + x69 +x79 + x89 >= 154.887;
* x310 + x410 +x510 + x610 +x710 + x810 >= 91.101;
* x311 + x411 +x511 + x611 +x711 + x811 >= 26.24;
* x312 + x412 +x512 + x612 +x712 + x812 >= 17.18;
* x313 + x413 +x513 + x613 +x713 + x813 >= 16.1;
* x314 + x414 +x514 + x614 +x714 + x814 >=15.326;
* x315 + x415 +x515 + x615 +x715 + x815 >= 14.2;
* x316 + x416 +x516 + x616 +x716 + x816 >= 8.268;
* x317 + x417 +x517 + x617 +x717 + x817 >= 5.403;
* x318 + x418 +x518 + x618 +x718 + x818 >= 3.874;
* x319 + x419 +x519 + x619 +x719 + x819 >= 3.442;
* x320 + x420 +x520 + x620 +x720 + x820 >= 3.391;
* x321 + x421 +x521 + x621 +x721 + x821 >= 3.267;
* x322 + x422 +x522 + x622 +x722 + x822 >= 3.198;
* x323 + x423 +x523 + x623 +x723 + x823 >= 3.19;

# Results

We ran our model in the LP solver to get an optimal solution and the following results were obtained:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| From | | To | | Path Selected | Amount Transported |
| 1 | Source | 2 | MCR | 1 | 393.998 |
| 2 | MCR | 3 | kaemari storage plant | 1 | 64.833 |
| 2 | MCR | 4 | shikarpur storage plant | 1 | 64.833 |
| 2 | MCR | 5 | mehmoodkot storage plant | 1 | 64.833 |
| 2 | MCR | 6 | sahiwal storage plant | 1 | 64.833 |
| 2 | MCR | 7 | machike storage plant | 1 | 64.833 |
| 2 | MCR | 8 | taru jaba storage plant | 1 | 64.833 |
| 3 | kaemari storage plant | 9 | karachi | 1 | 64.833 |
| 3 | kaemari storage plant | 10 | lahore | 0 | 0.000 |
| 3 | kaemari storage plant | 11 | faisalabad | 0 | 0.000 |
| 3 | kaemari storage plant | 12 | rawalpindi | 0 | 0.000 |
| 3 | kaemari storage plant | 13 | peshawar | 0 | 0.000 |
| 3 | kaemari storage plant | 14 | multan | 0 | 0.000 |
| 3 | kaemari storage plant | 15 | hyderabad | 0 | 0.000 |
| 3 | kaemari storage plant | 16 | islamabad | 0 | 0.000 |
| 3 | kaemari storage plant | 17 | sargodha | 0 | 0.000 |
| 3 | kaemari storage plant | 18 | sheikhupura | 0 | 0.000 |
| 3 | kaemari storage plant | 19 | rahim yar khan | 0 | 0.000 |
| 3 | kaemari storage plant | 20 | jhang | 0 | 0.000 |
| 3 | kaemari storage plant | 21 | dera ghazi khan | 0 | 0.000 |
| 3 | kaemari storage plant | 22 | gujrat | 0 | 0.000 |
| 3 | kaemari storage plant | 23 | sahiwal | 0 | 0.000 |
| 4 | shikarpur storage plant | 9 | karachi | 1 | 64.833 |
| 4 | shikarpur storage plant | 10 | lahore | 0 | 0.000 |
| 4 | shikarpur storage plant | 11 | faisalabad | 0 | 0.000 |
| 4 | shikarpur storage plant | 12 | rawalpindi | 0 | 0.000 |
| 4 | shikarpur storage plant | 13 | peshawar | 0 | 0.000 |
| 4 | shikarpur storage plant | 14 | multan | 0 | 0.000 |
| 4 | shikarpur storage plant | 15 | hyderabad | 0 | 0.000 |
| 4 | shikarpur storage plant | 16 | islamabad | 0 | 0.000 |
| 4 | shikarpur storage plant | 17 | sargodha | 0 | 0.000 |
| 4 | shikarpur storage plant | 18 | sheikhupura | 0 | 0.000 |
| 4 | shikarpur storage plant | 19 | rahim yar khan | 0 | 0.000 |
| 4 | shikarpur storage plant | 20 | jhang | 0 | 0.000 |
| 4 | shikarpur storage plant | 21 | dera ghazi khan | 0 | 0.000 |
| 4 | shikarpur storage plant | 22 | gujrat | 0 | 0.000 |
| 4 | shikarpur storage plant | 23 | sahiwal | 0 | 0.000 |
| 5 | mehmoodkot storage plant | 9 | karachi | 1 | 25.221 |
| 5 | mehmoodkot storage plant | 10 | lahore | 0 | 0.000 |
| 5 | mehmoodkot storage plant | 11 | faisalabad | 0 | 0.000 |
| 5 | mehmoodkot storage plant | 12 | rawalpindi | 0 | 0.000 |
| 5 | mehmoodkot storage plant | 13 | peshawar | 0 | 0.000 |
| 5 | mehmoodkot storage plant | 14 | multan | 1 | 15.326 |
| 5 | mehmoodkot storage plant | 15 | hyderabad | 1 | 14.200 |
| 5 | mehmoodkot storage plant | 16 | islamabad | 0 | 0.000 |
| 5 | mehmoodkot storage plant | 17 | sargodha | 0 | 0.000 |
| 5 | mehmoodkot storage plant | 18 | sheikhupura | 0 | 0.000 |
| 5 | mehmoodkot storage plant | 19 | rahim yar khan | 1 | 3.442 |
| 5 | mehmoodkot storage plant | 20 | jhang | 1 | 3.377 |
| 5 | mehmoodkot storage plant | 21 | dera ghazi khan | 1 | 3.267 |
| 5 | mehmoodkot storage plant | 22 | gujrat | 0 | 0.000 |
| 5 | mehmoodkot storage plant | 23 | sahiwal | 0 | 0.000 |
| 6 | sahiwal storage plant | 9 | karachi | 0 | 0.000 |
| 6 | sahiwal storage plant | 10 | lahore | 1 | 30.142 |
| 6 | sahiwal storage plant | 11 | faisalabad | 1 | 26.240 |
| 6 | sahiwal storage plant | 12 | rawalpindi | 0 | 0.000 |
| 6 | sahiwal storage plant | 13 | peshawar | 0 | 0.000 |
| 6 | sahiwal storage plant | 14 | multan | 0 | 0.000 |
| 6 | sahiwal storage plant | 15 | hyderabad | 0 | 0.000 |
| 6 | sahiwal storage plant | 16 | islamabad | 0 | 0.000 |
| 6 | sahiwal storage plant | 17 | sargodha | 1 | 5.247 |
| 6 | sahiwal storage plant | 18 | sheikhupura | 0 | 0.000 |
| 6 | sahiwal storage plant | 19 | rahim yar khan | 0 | 0.000 |
| 6 | sahiwal storage plant | 20 | jhang | 1 | 0.014 |
| 6 | sahiwal storage plant | 21 | dera ghazi khan | 0 | 0.000 |
| 6 | sahiwal storage plant | 22 | gujrat | 0 | 0.000 |
| 6 | sahiwal storage plant | 23 | sahiwal | 1 | 3.190 |
| 7 | machike storage plant | 9 | karachi | 0 | 0.000 |
| 7 | machike storage plant | 10 | lahore | 1 | 60.959 |
| 7 | machike storage plant | 11 | faisalabad | 0 | 0.000 |
| 7 | machike storage plant | 12 | rawalpindi | 0 | 0.000 |
| 7 | machike storage plant | 13 | peshawar | 0 | 0.000 |
| 7 | machike storage plant | 14 | multan | 0 | 0.000 |
| 7 | machike storage plant | 15 | hyderabad | 0 | 0.000 |
| 7 | machike storage plant | 16 | islamabad | 0 | 0.000 |
| 7 | machike storage plant | 17 | sargodha | 0 | 0.000 |
| 7 | machike storage plant | 18 | sheikhupura | 1 | 3.874 |
| 7 | machike storage plant | 19 | rahim yar khan | 0 | 0.000 |
| 7 | machike storage plant | 20 | jhang | 0 | 0.000 |
| 7 | machike storage plant | 21 | dera ghazi khan | 0 | 0.000 |
| 7 | machike storage plant | 22 | gujrat | 0 | 0.000 |
| 7 | machike storage plant | 23 | sahiwal | 0 | 0.000 |
| 8 | taru jaba storage plant | 9 | karachi | 0 | 0.000 |
| 8 | taru jaba storage plant | 10 | lahore | 0 | 0.000 |
| 8 | taru jaba storage plant | 11 | faisalabad | 0 | 0.000 |
| 8 | taru jaba storage plant | 12 | rawalpindi | 1 | 17.180 |
| 8 | taru jaba storage plant | 13 | peshawar | 1 | 16.100 |
| 8 | taru jaba storage plant | 14 | multan | 0 | 0.000 |
| 8 | taru jaba storage plant | 15 | hyderabad | 0 | 0.000 |
| 8 | taru jaba storage plant | 16 | islamabad | 1 | 8.268 |
| 8 | taru jaba storage plant | 17 | sargodha | 1 | 0.156 |
| 8 | taru jaba storage plant | 18 | sheikhupura | 0 | 0.000 |
| 8 | taru jaba storage plant | 19 | rahim yar khan | 0 | 0.000 |
| 8 | taru jaba storage plant | 20 | jhang | 0 | 0.000 |
| 8 | taru jaba storage plant | 21 | dera ghazi khan | 0 | 0.000 |
| 8 | taru jaba storage plant | 22 | gujrat | 1 | 3.198 |
| 8 | taru jaba storage plant | 23 | sahiwal | 0 | 0.000 |

The above table represents that how much fuel is moving from one node to another while fulfilling the demand requirements of each final node with minimum cost.

# Glossary

The node represents following areas/ storages:

1: Sources (PPL Hala, JJVL Jamshoro, OGDCL Bobi and Kunnar)

2: Mid-Country Refinery (MCR)

3: Kaemari storage plant

4: Shikarpur storage plant

5: Mehmoodkot storage plant

6: Sahiwal storage plant

7: Machike storage plant

8: Taru Jaba storage plant

9: Karachi

10: Lahore

11: Faisalabad

12: Rawalpindi

13: Peshawar

14: Multan

15: Hyderabad

16: Islamabad

17: Sargodha

18: Sheikhupura

19: Rahim Yar Khan

20: Jhang

21: Dera Ghazi Khan

22: Gujrat

23: Sahiwal

# References:

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